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Lecture 17: Deleuze and the Calculus

Introduction

In this lecture, I just want to give a basic account of Deleuze's use of the differential calculus in chapter four of *Difference and Repetition*. As we'll see next week, the calculus provides a model for the way in which we understand Deleuze's conception of the Idea. As such, it is the cornerstone of his alternative to the notion of representation we looked at earlier in the term. Before moving on to the calculus itself, I just want to recap some of the factors motivating his account of Ideas that we've come across over the course.

First, there was the discussion of incongruent counterparts we looked at in the introduction of *Difference and Repetition*. There we saw that Deleuze opposed both Kant and Leibniz's account of the nature of space. As we saw, Deleuze took up Kant's claim that certain properties such as handedness could not be understood in conceptual terms. The implication of this is that any account that sees the object as completely determined by *conceptual* determinations has to be false, as such an account will not be able to explain all of the properties that objects have. Nonetheless, Deleuze also rejected Kant's claim that, while objects of intuition had properties that were non-conceptual, these properties were purely reliant on the nature of *given* space. That is, Deleuze argued that what was needed was a genetic enquiry that explained why space made incongruent counterparts possible. Now, an implication of this is that if we are going to explain what makes space possible, the terms we use for this description cannot themselves be understood in spatial terms, otherwise our explanation of space would be empty, or lead us to an infinite regress. This gave us our initial, negative account of Deleuze's project: he was seeking the genetic conditions for objects, where such genetic conditions were both non-conceptual (as Kant's argument shows), and non-spatial.

Second, at the beginning of this term, we looked at Deleuze's critique of the image of thought. Deleuze's claim was that the way in which we normally understand the world relies on two basic structures: good sense (the self in general terms) and common sense (the kind of hierarchy we found in Porphyry). Now, following Feuerbach, Deleuze argued that these structures emerge not as a result of thinking itself, but as a result of communication. In order for me to present my thought to another, I need to transform it into a successive structure (because I can't say everything at once), and I need to abstract those aspects of thinking that derive from my own particularity of thinking. One of the aims of the chapter on the image of thought was therefore to provide an account of how thinking emerged as structured in temporal and abstract conceptual terms. As we saw, Deleuze presented this approach in the language of problems and solutions. Thinking was a problem that led to the solution of the image of thought. The challenge was to come up with a way of thinking problems that did not derive them from solutions (or to think the grounds of propositional thought in a way that didn't just presuppose them). So the project of chapter three of *Difference and Repetition* mirrors that which Deleuze presents earlier in the book. In both cases, what we are looking for is a non-spatiotemporal non-conceptual ground for objects, or our categories of thinking about objects. When we turn to the calculus in a minute, we'll see that this kind of account is what Deleuze is hoping to provide.

The final point that I want to return to is Kant's account of Ideas that we looked at last week. As we saw, Kant's claim was that while the relationship between judgement and intuition could give us particular facts about the world, in order to actually develop knowledge, we needed to move beyond this, and synthesise these individual facts into a system of knowledge. This process involved constructing greater and greater unities of concepts, similar to the way in which we can trace back definitions for Aristotle to more and more general terms. Now, Kant's claim was that in order for reason to carry out the task of unifying knowledge, it needed the Idea of an absolute unity or totality to act as what he calls as *focus imaginarius* for the process itself. The totalities: the soul, the world, and God, go beyond what is available to experience, and hence, while we can *think* these notions, we cannot *know* them. Given that we simply do not know whether or not, for instance, God exists, Kant calls these Ideas 'problematic.' Now, the Kantian Ideas appear to meet all of the criteria for the kind of genetic account we were looking for. First, they make thinking possible. While they aren't *transcendental* conditions of the possibility of experience in the strict sense, without them any form of systematic thought would be impossible. Second, insofar as they are problematic, they are not determined by any of the categories of experience, either conceptual or spatio-temporal:

> The object of an Idea, Kant reminds us, is neither fiction nor hypothesis nor object of reason: it is an object which can be neither given nor known, but must be represented without being able to be directly determined. Kant likes to say that problematic Ideas are both objective and undetermined. The undetermined is not a simple imperfection in our knowledge or a lack in the object: it is a perfectly positive, objective structure which acts as a focus or horizon within perception. (DR 215)

Now, as we saw, Deleuze ultimately rejected this account as an account of the genesis of thought as, while the Idea was undetermined, when it came to determining it, or bringing it into representation, its determinate aspects were taken from experience, rather than giving rise to experience. That is, the Idea was brought into experience by representing it by analogy with objects that we do experience. Taking the notion of the ground of all appearances, for instance – we define it as God by analogy with experience. Similarly, God allows us to see objects of experience as completely determined by properties, but this is only on the basis of seeing God himself as an object like those we find in experience: 'The Ideal is, therefore, the archetype (*prototypon*) of all things, which one and all, as imperfect copies (*ectypa*), derive from it the material of their possibility, while approximating to it in various degrees.' (CPR A578/B606)

So in these respects, Ideas do not really provide problems that are definable apart from their solutions. While they are undetermined, making them determinable and determining them relies on an analogy with objects of experience (this is what Deleuze means by two of the Ideas characteristics being extrinsic). If we are going to explain how representation becomes possible, it can't simply be the case that Ideas are *brought into* representation (by analogy), but rather that they give rise to it. That is, they must be determinable within space and time intrinsically. Deleuze thinks that the calculus provides a model for how this is possible.

Now, at this point, we need to make a distinction between the mathematics of the calculus, and the interpretation of how that mathematics operates. I'll follow Leibniz's interpretation in giving an account of how it works, but as we'll see, Deleuze has a different interpretation of what the terms mean.

The Calculus

If we represent on a graph the relation between distance travelled and time taken to travel that distance by an object, it becomes possible to determine the velocity of the object by dividing the distance travelled by the time, hence velocity is measured in terms such as miles per hour, metres per second, etc. If the velocity is constant, the relation between distance and time will be proportional. This means that the line representing the moving object will be straight. If, therefore, we wish to determine the velocity of the object, we simply need to take a section of the line, and divide the distance travelled over that time, which will be represented by the change in the value on the distance axis over the length of the section, by the time, which will be represented by the change in the value of the time axis over the length of the section. As the two values are directly proportional, a section of any arbitrary length will provide the same result.

if we are dealing with an object moving at a velocity which is not constant, then this procedure cannot be used, as we were able to determine the velocity at any point using the previous method only because the velocity was the same at every point (the average velocity is the same as the velocity at every instant). Instead, however, we can measure the velocity at any point of a system with a changing velocity by drawing a graph of the function of this change, that is, of the relation of distance to time, and drawing a line which runs parallel to a particular point on the curve. This produces a vector of the velocity of the system at this particular moment. The difficulty with this approach is that it can only be approximate, as we are attempting to draw a line through a point, which in itself can seemingly have no direction. The alternative, to draw a line through two points of the curve, is equally flawed, as although it gives us an accurate line, we are dealing with a curve, and so the tangent we are now drawing will not represent the velocity at one particular moment, but the average between the two points. Leibniz's solution to this difficulty was to draw a line between the point whose velocity we wish to measure, and another arbitrary point on the curve, and then to imagine the distance between these two points decreasing towards zero. As we now have a straight line between these two points, we can treat the case in the same manner as the case of constant velocity described above, measuring the change in values of both axes along a length of the line. Thus mathematically, we end up with two lines, one representing the change over the section in terms of distance, and one in terms of time, neither of which on its own will have any determinate value, as the lines are infinitely short, but when divided, one by the other, will give a vector at the particular point. Since the axes of the graph can represent more than simply time and distance, these values are referred to more generally as *dy* and *dx*. So if we are given a mathematical function, we can work out the formula for the gradient at any point by differentiating it. An important point is that this process is reversible – that is, we can move from an equation in terms of *dy*/*dx* to the equation it gives the gradient for (known as the primitive function) by a process of integration.

Deleuze's Interpretation of the Calculus

Now, on this basis, we can already start to see why Deleuze thinks that the differential calculus provides a model for the Idea. Central to this account is going to be the differential itself. In this regard, the following claim is essential: 'Just as we oppose difference in itself to negativity, so we oppose dx to not-A, the symbol of difference [Differenzphilosophie] to that of contradiction.' (DR 217) If we return to the three moments that Deleuze argued needed to be intrinsically related for an account of the genesis of representational thought, we can see that they can all be mapped onto the calculus. Deleuze thinks that we can show that these three moments can be mapped onto determinations in terms of quantity, quality, and potentiality, but as I just want to give you an idea of what he's talking about, I want to focus on quantity. In terms of mathematics, this just means whether an equation or element in an equation has a determinate value or not (so the gradient of an actual point on a curve is a definite number, and so has a quantitative determination). We have been dealing with the calculus as a method of going from a curve to its gradient, but Deleuze reverses the direction of analysis. He is rather interested in the way in which the determinate values of the curve can be generated from the indeterminate nature of the differential.

As I said earlier, each of the values, *dy* and *dx* is on its own completely lacking in magnitude. 'dx is strictly nothing in relation to x, as dy is in relation to y.' (DR 218) This is because what gives us the gradient to a curve isn't the differentials themselves, but rather the ratio between them. If the terms, dy and dx are understood outside of this ratio, then, as they represent infinitesimal lengths, they have no determinate magnitude. Now, Deleuze's claim is that whilst *dx* is strictly nothing in relation to *x*, this is not because the differential is not in a sense real, but rather because it cannot be captured by either (Kantian) intuition or the categories of quantity. In this sense, the differential, *dx*, as a symbol of difference, is 'completely undetermined', that is, as the representation of the 'closest noumenon', difference, it escapes the symbolic order. The symbols, *dy* and *dx*, and their values of 0 in respect to *y* and *x*, therefore represent the annihilation of the quantitative within them in favour of what Deleuze calls the sub-representational, or extra-propositional. As such, Deleuze's claim is that it is this element that represents the original thought of Feuerbach.

If differentials are indeterminate by themselves, they are nonetheless determinable. When differentials are brought into relation with one another, they together determine a curve that *is* determinable. That is, the function, *dy/dx* allows us to work out the gradient at any point on the curve. Now, *dy/dx* is a ratio, but it is a very particular kind of ratio. In most ratios, such as ¾ or 1/2, the terms that make it up have a determinate quality apart from the determinate quality of the ratio itself. Even a ratio such as y/x is one where the elements, y and x just stand in for determinate values. 'The relation dy/dx is not like a fraction which is established between particular quanta in intuition, but neither is it a general relation between variable algebraic magnitudes or quantities. Each term exists absolutely only in its relation to the other: it is no longer necessary, or even possible, to indicate an independent variable.' (DR 219)

Finally, the function, *dy/dx* doesn't simply give us a function for the gradient at any point on the curve, but we also have the specific value of the gradient or ratio when we solve the differential for specific values of x and y. This final moment, where we are dealing with a precise value, gives us the equivalent of complete determination for a point on the curve.

So while only one of the moments was an intrinsic feature of the Idea for Kant, the differential calculus, for Deleuze, provides a model where all three aspects are intrinsic features. Thus, we have the differential, dx, as an element that is simply incapable of being represented. Nonetheless, by entering into a relationship with another differential, both become reciprocally determined, giving an equations for generating the gradients at points on the curve. We also have the specific points that are determined (complete determination). Thus, we have a model that takes us from a non-spatial, non-conceptual moment, through a process of reciprocal determination, to a determinate extension, which provides just the model Deleuze was looking for in his critiques of Kant and Leibniz. It should be clear that the dx, for Deleuze, is the moment of thinking prior to representation that Feuerbach was looking for, and that Deleuze thought Kant was missing. What are the moments of the equation and the specific value? Well, Deleuze presents them as essentially the concept of an object (determinability), and the specific object the concept relates to (complete determination). Just as the concept, man, relates to all of the different varieties of men, the equation allows us to specify the gradient at each point on the curve.

While this model might appear quite abstract at the moment, next week we'll look at a more concrete case – the way we understand the structure of organic life – which shows why the approach Deleuze is suggesting here might provide an alternative to Aristotelian accounts of determination, as well as explaining some central aspects of the way in which life operates. For the remainder of the lecture, I want to deal with one issue I have left to one side until now – the relationship between Leibniz's account of the calculus (the account I've given you), and the problem/solution complex which we looked at earlier in the term.

Problems, Solutions, and the Calculus

The difficulty with Kant's account of Ideas was that, while he recognised that Ideas were problematic, in actual fact, two of the essential aspects of the Idea were determined by experience itself. Thus, in Deleuze's terms, the problem which gives rise to experience wasn't properly specified, because it was understood in terms of the solution (experience itself). Now when we look at Leibniz's account of the calculus, it appears that the same thing is happening. That is, when Leibniz wishes to define what a differential is, he begins with the notion of sensible experience (a finite ratio), and then imagines what would happen if we shrank it so it became infinitesimal. If the differential is understood in these terms, then it would seem to be the case that we haven't really succeeded in defining a problem independently of its solutions at all. That is, we start from experience to define the infinitesimal that is then used to explain experience. Along with a reading of how the calculus functions, Deleuze also needs a new reading of the foundations of the calculus, such that differentials aren't understood in terms of sense experience. Now, as we'll see, Leibniz's account of the foundations of the calculus is in fact contradictory, leading to Deleuze positing a new account of the foundations.

To recap, Leibniz's method relies on the infinitesimal, an infinitely small difference between two points, *dy*. As this difference is infinitely small, it can be discounted for the purposes of calculation, but, as it retains a magnitude relative to *dx*, it could be used to form a ratio, *dy*/*dx* which had a determinate value. When we look at the first postulate of L'hôpital's *Analyse*, which provided the first general introduction to the method, we can see immediately that Leibniz's approach appears problematic:

> *Postulate I*. Grant that two quantities, whose difference is an infinitely small quantity, may be taken (or used) indifferently for each other: or (which is the same thing) that a quantity, which is increased or decreased only by an infinitely smaller quantity, may be considered as remaining the same.

It is this postulate which allows us to discount the effects of *dx* in the results of our derivation of the differential function. It also leads to the problems of the foundations of the calculus which, although known to Leibniz and Newton, were made notorious in Berkeley's treatise, *The Analyst*. The essence of Berkeley's criticism lies in the assertion that:

> If with a view to demonstrating any proposition a certain point is supposed, by virtue of which certain other points are attained; and such supposed point be itself afterwards destroyed or rejected by a contrary supposition; in that case, all other points, attained thereby and consequent thereupon, must also be destroyed and rejected, so as from thence forward to be no more supposed or applied in the demonstration. This is so plain as to need no proof.

Berkeley's essential point is that it is contradictory to assume both that the differential has a magnitude (so it can form a ratio), and that it does not (so it can be the gradient at a point). The contradictory nature of these two characteristics led to a massive amount of metaphysical speculation in the nineteenth century aimed at explaining how the calculus works.

Deleuze's claim is that Leibniz goes wrong by failing to recognise the significance of the break between the infinitesimal value and the ratio. Leibniz sees simply a difference in degree between the infinitesimal and determinate magnitudes. Deleuze rather claims that there is a difference in kind between the two: 'In short, the limit must be conceived not as the limit of a function but as a genuine cut [coupure], a border between the changeable and the unchangeable within the function itself.' (DR 218) The infinitesimal is not simply infinitely small, but is sub-representational. It generates representations by being brought into relation with others. It's worth noting that the claim here that Leibniz makes a mistake by interpreting the differential as only different in degree from the representations it generates is very similar to the claim about Leibniz Deleuze made in the introduction of *Difference and Repetition*. There we saw that Deleuze's criticism of Leibniz's metaphysics was that there was only a difference in degree between the concept of the monad and the world of experience it generated, as both were ultimately understood in purely conceptual terms.

Conclusion

To conclude, I think there are two themes running through this analysis that I want to pick up on over the next few weeks.

First, that problems are the grounds for solutions. We can see in his criticisms of Kant, Descartes, and Plato, that the essential heart of his criticisms are that we move from solutions (experience) to problems, rather than vice-versa.

Second, the structure of solutions differs from that of problems. We'll go into this in more detail next week, but we can already not that whereas objects are determined through opposition (the rational/nonrational distinction, for instance), differentials are determined by entering into reciprocal relations with one another.

Next week, we'll see how this scheme works in a more empirical context.